**647510** **BSc5MatC503x** **Seat No :________**

# B.Sc. Semester - 5 (*CBCS*) Examination
# Oct/Nov - 2024 (NEW COURSE)
# Boolean Algebra & Complex Analysis -1(Core)

**Time: 2:30 Hours** **Marks: 70**

**Instructions:**

1. All questions are compulsory.
2. Figures to the right indicate marks.

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Que.1(A) Answer any two. (10)

1. R is an equivalence relation on set X then prove that
   (i) $\cup_{x \in X}[x]$ = X
   (ii) $x$ R $y$ $\Leftrightarrow [x] = [y]$
2. State and prove distributive inequalities.
3. Let (L, $*$, $\oplus$ ) be a Lattice, a, b, c $\in$ L then prove that
   $a * (b \oplus c) = (a * b) \oplus (a * c) \Leftrightarrow a \oplus (b * c) = (a \oplus b) * (a \oplus c)$

Que.1(B) Answer any one. (04)

1. In usual notation show that $(S_{30}$, D) is Poset.
2. A = {a, b, c} and $\subseteq$ is a relation of subset or equal set then show that (P(A), $\subseteq$) is Poset.

Que.2(A) Answer any two. (10)

1. If (B, $*$ , $\oplus$ , ' , 0, 1) is a Boolean algebra and $x_1$, $x_2$ $\in$B then prove that
   $A(x_1 * x_2) = A(x_1) \cap A(x_2)$.
2. Obtain sum of product canonical form of $\alpha(x_1, x_2, x_3) = x_1x_3 \oplus x_2x_3'$ .
3. If S = {a, b, c}, B = {0, 1} then show that $f: P(S) \rightarrow B$ is Boolean homomorphism from $(P(S), \cap, \cup, {}^c, \phi, S)$ to $(B, *, \oplus, ', 0, 1)$ where
   $f(A) = \begin{cases} 1 \quad , & b \in A \\ 0 \quad , & b \notin A \end{cases}$ for all $A \in P(S)$ .

Que.2(B) Answer any one. (04)

1. For Boolean statement prove that
   $(a' * b' * c) \oplus (a * b' * c) \oplus (a * b' * c') = b' * (a \oplus c)$
2. If (B, $*$ , $\oplus$, ', 0, 1) is Boolean algebra and a, b $\in$ B then prove that
   $a \leq b \Leftrightarrow a * b' = 0 \Leftrightarrow b' \leq a' \Leftrightarrow a' \oplus b = 1$.

Que.3(A) Answer any two. (10)

1. In usual notation obtain Cauchy – Riemann conditions in Cartesian form.
2. Prove that $f(z) = \begin{cases} \frac{(\bar{z})^2}{z} , & z \neq 0 \\ 0 , & z = 0 \end{cases}$ satisfies C – R conditions at origin, however $f(z)$ is not analytic at origin.
3. Find an analytic function $f$ (z) = u + i v such that u – v = x + y.

Que.3(B) Answer any one. (04)

1. Prove that u = $r^2\cos 2\theta$ is a harmonic function and also find it's conjugate.
2. Obtain Laplace equation in polar form.

Que.4(A) Answer any two. (10)

1. State and prove Cauchy's fundamental theorem of integration.
2. Find $\int_c \frac{z}{(z-2)\left(z+\frac{1}{2}\right)}$ dz; where C: $|z|$ = 1.
3. Show that $\left|\int_c \frac{z+4}{z^3-1}\, dz\right| \leq \frac{6\pi}{7}$ where C be the arc of the circle $|z|$ = 2 from z = 2 to z = 2i.

Que.4(B) Answer any one. (04)

1. Find $\int_c \frac{z}{(9-z^2)(z+i)}$ dz; where C: $|z|$ = 2.
2. Find: $\int_c \frac{dz}{z^2+4}$ where C: $|z-i| = 2$.

Que.5(A) Answer any two. (10)

1. State and prove the fundamental theorem of Algebra.
2. State and prove Liouville's theorem.
3. State and prove Morera's theorem.

Que.5(B) Answer any one. (04)

1. Find: $\int_c \frac{z^2+3}{z^2(z-4)}$ dz where C: |z| = 1.
2. Find: $\int_c \frac{z^3+2}{(z+i)^3}\, dz$ where C: | z | = 2.

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